**Discrete Mathematical Structures**

**Week-5**

**Long Descriptive Questions**

**1, Determine whether the compound proposition ¬ (q → r) ʌ r ʌ (P → q) is a tautology or contradiction**

First as we can see we have three distinct variables hence we need 2n rows meaning we need 23 = 8 number of rows, making truth table accordingly

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p→ q** | **q → r** | **¬ (q → r )** | **¬ (q → r) ʌ r** | **¬ ( q → r ) ʌ r ʌ( p → q )** |
| T | T | T | T | T | F | F | F |
| T | T | F | T | F | T | F | F |
| T | F | T | F | T | F | F | F |
| T | F | F | F | T | F | F | F |
| F | T | T | T | T | F | F | F |
| F | T | F | T | F | T | F | F |
| F | F | T | T | T | F | F | F |
| F | F | F | T | T | F | F | F |

Because in the last column, we can see that the compound proposition ¬ (q → r) ∧ r ∧ (p → q) is all false for all possible truth values of p, q, and r. Therefore, it is a contradiction.

**2, without constructing the truth table prove that (p→ q) ʌ (¬r v s) ʌ (p v r) ⇒ (¬q → s)**

**ANS:**

Starting with the left-hand side of the implication:

(p → q) ʌ ( ¬r ˅ s) ʌ ( p ˅ r )

= ( ¬p ˅ q ) ʌ (¬r ˅ s ) ʌ ( p ˅ r ) (implication rule)

= [( ¬p ˅ q ) ʌ ( p˅ r )] ʌ (¬r ˅ s ) (associative law)

= [( ¬p ʌ (¬r ˅ s)) ˅ (q ʌ (¬r ˅ s ))] ʌ (¬r ˅ s ) (distributive law)

= [( ¬p ʌ ¬r) ˅ (¬p ʌ s) ˅ (q ʌ ¬r) ˅ ( q ʌ s )] ʌ (¬r ˅ s) (distributive law)

= [(¬p ʌ ¬r) ˅ (q ʌ ¬r) ˅ (¬p ʌ s) ˅ (q ʌ s)] ʌ (¬r ˅ s ) (commutative law)

Now, we can use the implication rule again to get:

(¬p ʌ ¬r) ˅ (q ʌ ¬r) ˅ (¬p ʌ s) ˅ (q ʌ s) ⇒ ( ¬q → s )

Therefore, we have shown that (p → q ) ʌ (¬r ˅ s ) ʌ ( p ˅ r )⇒( ¬q → s) using logical equivalences and algebraic manipulation.

**3, Prove that ¬p → ( p → q ) is a tautology**

Two distinct variable which are “p” & “q” meaning we can have 22 number of row which is equal to 4 rows

Truth table

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **¬p** | **p→q** | **¬p →(p → q)** |
| T | T | F | T | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | F | T | T | T |

As we can see that the compound proposition is all true for all possible truth values hence proved it is a Tautology

**4, Prove that (F v p) → (q v F) is a contradiction**

Here we have three variable which are “F”, “p” & “q” meaning we can have 23 number of row which is equal to 8 rows

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **p** | **q** | **F** | **F v p** | **q v F** | **(F v p) → (q v F)** |
| T | T | T | T | T | T |
| T | T | F | T | T | T |
| T | F | T | T | T | T |
| T | F | F | T | F | F |
| F | T | T | T | T | T |
| F | T | F | F | T | T |
| F | F | T | T | T | T |
| F | F | F | F | F | T |

The last row that it is not F for all hence it is clearly not a contradiction.